# Microwave Stepped Impedance Filter Design Sheet

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TThe maths for this can be found at www.acusd.edu/~ekim/e194rfs01/iec19aek.pdf. I have also added some simulation results at the end.

There are three steps to this filter design:

**Step 1**, Get the filter g-values, this sheet calculated the g values according to the Chebychev polynomial, but these can be obtained in a variety of ways from books or from a program freely available from my web page. You will be pleased to know that these g-values agree with my program!!

**Step 2**, Calculate Zo and wavelength depending on the type of filter.
This MathCAD sheet stops at this point.

**Step 3**, Calculate length and width. One method is to use Linecalc which is part of ADS. For microstrip designs I have written a MathCAD sheet which is available from my wepage that works well for thin tracks, less well for thicker tracks. Also you can try **transcalc.sourceforge.net** for a Linecalc equivalent.

Yellow is user input, Green is output

## Main user input area:

$\mu m := 10^{-6} \cdot m \qquad nH := 10^{-9} \cdot henry$

$L_{ar\_db} := 0.2$ passband ripple in dB

$N := 7$ order of the filter

$f_c := 1 \cdot GHz$ Cutoff frequency.

$Z_o := 50 \cdot \mathbf{O}$

### LPF Frequency Response, for Chebychev Polynomials

This section plots the frequency response assuming the Chebychev polynomial values calculated later. The frequency response is for the coupled BPF, the LPF response is later.

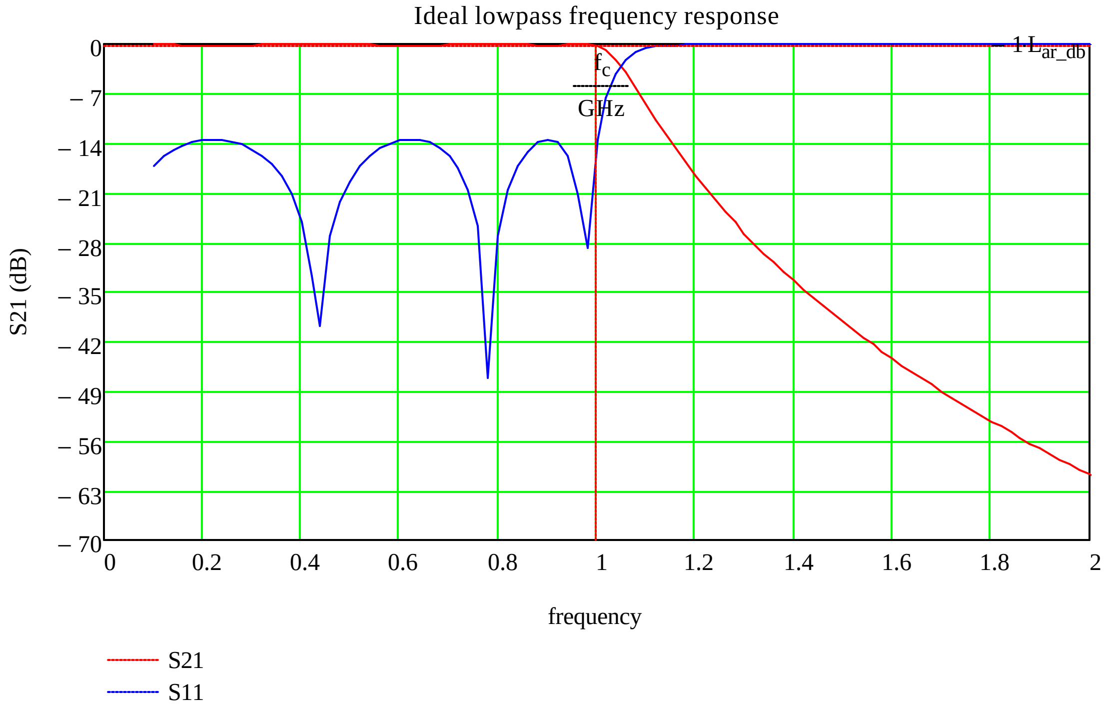
$$Y_o := \frac{1}{Z_o} \qquad \mathbf{e} := 10^{\frac{L_{ar\_db}}{10}} - 1$$

$\mathbf{e} = 0.047$

$$L_A(f, f_1) := \begin{cases} 10 \cdot \log\left[1 + \mathbf{e} \cdot \left[\cos\left(\left(N \cdot \operatorname{acos}\left(\frac{f}{f_1}\right)\right)\right)\right]^2\right] & \text{if } f \le f_1 \\ 10 \cdot \log\left[\left[1 + \mathbf{e} \cdot \left[\cosh\left(\left(N \cdot \operatorname{acosh}\left(\frac{f}{f_1}\right)\right)\right)\right]^2\right]\right] & \text{if } f > f_1 \end{cases}$$

$$S11_A(f, f_1) := 10 \cdot \log\left[1 - 10^{\left(\frac{-L_A(f, f_1)}{10}\right)}\right]$$

$$f_{lp\_hp\_sweep\_wide} := \frac{f_c}{10}, \frac{f_c}{10} + \frac{f_c}{50} .. f_c \cdot 5 \qquad f_{lp\_hp\_sweep\_narrow} := \frac{f_c}{10}, \frac{f_c}{10} + \frac{f_c}{100} .. f_c \cdot 1.15$$




## Calculate the Chebychev (g) Polynomials

$k := 1..N$ $\qquad$ $ß := \ln\left(\coth\left(\frac{L_{ar\_db}}{17.37}\right)\right)$ $\qquad$ $? := \sinh\left(\frac{ß}{2 \cdot N}\right)$ $\qquad$ $a_k := \sin\left[\frac{(2 \cdot k - 1) \cdot \mathbf{p}}{2 \cdot N}\right]$

$b_k := ?^2 + \left(\sin\left(\frac{k \cdot \mathbf{p}}{N}\right)\right)^2$ $\qquad$ $g_k := 0$ $\qquad$ $g_0 := 1$ $\qquad$ $g_{N+1} := 1$

g(0) and g(N+1) represent the input/output coupling for odd order filters, these are 1 representing the generator (and equal) load resistance

$$g_k := \begin{cases} \frac{2 \cdot a_1}{?} & \text{if } k = 1 \\ \frac{4 \cdot a_{k-1} \cdot a_k}{b_{k-1} \cdot g_{k-1}} & \text{otherwise} \end{cases}$$

$$g = \begin{pmatrix} 1 \\ 1.372 \\ 1.378 \\ 2.276 \\ 1.5 \\ 2.276 \\ 1.378 \\ 1.372 \\ 1 \end{pmatrix}$$

## The next step is to calculate Zo.

This next section converts the g-values to Zo-values. If you wish to use your own g-values, i.e not from the Chebychev polynomial then this is ok and you need to put them here as shown on the right. User sets Zlow and Zhigh based on what is reasonable for the substrate.

**This filter alternates between these impedances**

$g_{100} := 2$

$Z_{low} := 20$ $\qquad$ $Z_{high} := 120$

$$angle\_L\_equiv_k := \operatorname{asin}\left(\frac{g_k \cdot \frac{Z_o}{\mathbf{O}}}{Z_{high}}\right) \qquad angle\_C\_equiv_k := \operatorname{asin}\left[\frac{Z_{low}}{\frac{\left(\frac{Z_o}{\mathbf{O}}\right)}{g_k}}\right]$$

There are a number of points to note:

1. The user needs to choose Zhigh/Zlow depending on the substrate.
2. If the angle exceeds 90 degrees then the ripple and N should be changed.
3. For max repeat frequency then the smaller the maximum angle is better.
4. The filter is designed by switching between the inductance/capacitance values in turn, hence two filters are possible from any given results.

| angle_L_equiv$_k$ = | | angle_C_equiv$_k$ = | |
|---|---|---|---|
| 34.875 | · deg | 33.293 | · deg |
| 35.047 | | 33.455 | |
| 71.478 | | 65.543 | |
| 38.687 | | 36.874 | |
| 71.478 | | 65.543 | |
| 35.047 | | 33.455 | |
| 34.875 | | 33.293 | |

## Calculate dimensions

Now we can calculate the length of each section.

$\mathbf{e}_r := 4.5$ $\quad ?_0 := \frac{3 \cdot 10^8 \cdot \frac{m}{s}}{f_c}$ $\quad ? := \frac{?_0}{\sqrt{\mathbf{e}_r}}$

$? = 141.421 \cdot mm$

$$length\_L\_equiv_k := ? \cdot \frac{angle\_L\_equiv_k}{360 \cdot deg} \qquad length\_C\_equiv_k := ? \cdot \frac{angle\_C\_equiv_k}{360 \cdot deg}$$

| length_L_equiv$_k$ = | | length_C_equiv$_k$ = | |
|---|---|---|---|
| 13.7 | · mm | 13.079 | · mm |
| 13.768 | | 13.142 | |
| 28.079 | | 25.748 | |
| 15.198 | | 14.486 | |
| 28.079 | | 25.748 | |
| 13.768 | | 13.142 | |
| 13.7 | | 13.079 | |

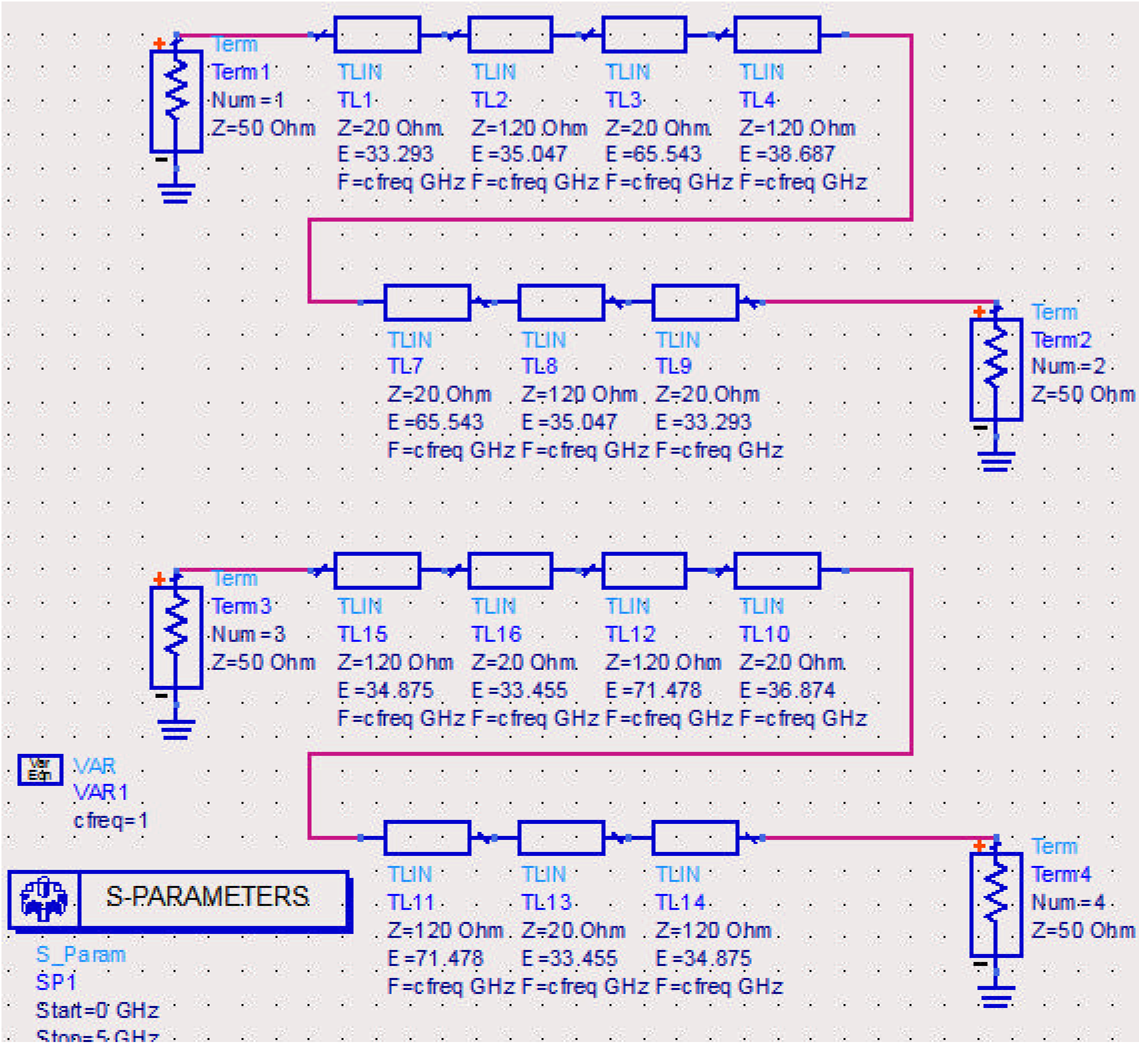
## Simulation Results

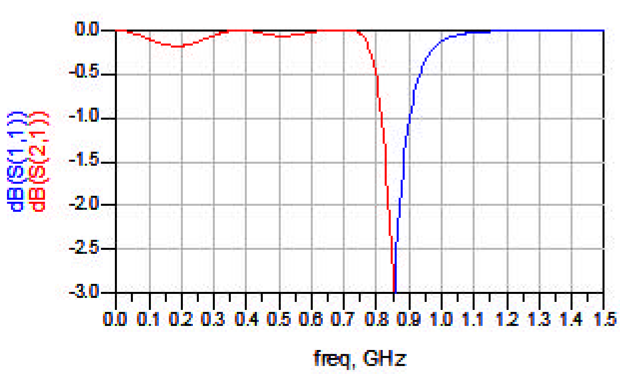
On the next page you can see somje simulation results

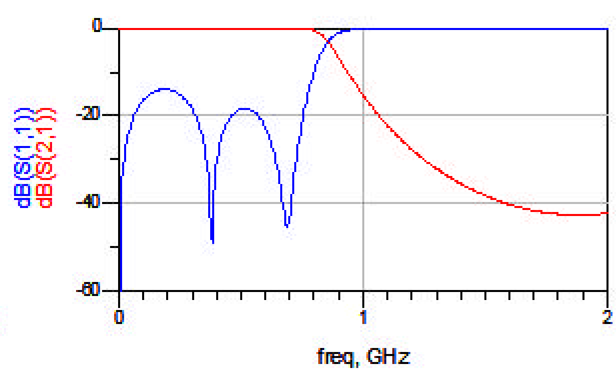
This algorithm makes two filters, one filter starts with Lequivalent then Cequivalent and the other filter is the other way around.

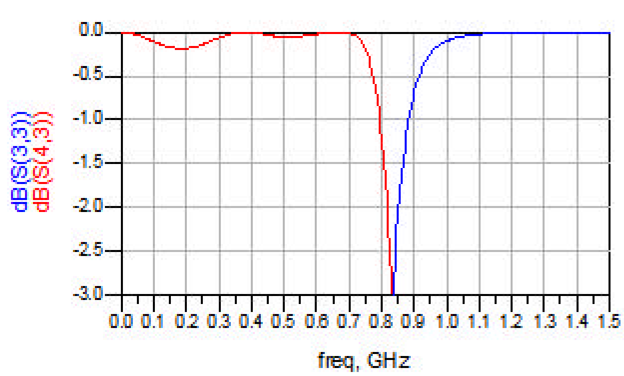
To test these calculations I have run a simulation on ADS, but with angles and not lengths. I want to see the basic algorithm works, which it does.

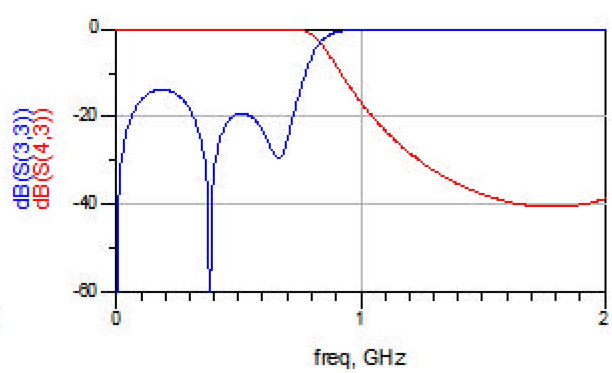
As you can see the results are in good agreement. Both the ripple and return loss (S11) agree, but the cutoff frequency, here normalised to 1 is out slightly and I am not sure why this is. Both types of filter agree with each other.


Term
Term1
Num=1
Z=50 Ohm
TLIN
TL1
Z=20 Ohm
E=33.293
F=cfreq GHz
TLIN
TL2
Z=120 Ohm
E=35.047
F=cfreq GHz
TLIN
TL3
Z=20 Ohm
E=65.543
F=cfreq GHz
TLIN
TL4
Z=120 Ohm
E=38.687
F=cfreq GHz
TLIN
TL7
Z=20 Ohm
E=65.543
F=cfreq GHz
TLIN
TL8
Z=120 Ohm
E=35.047
F=cfreq GHz
TLIN
TL9
Z=20 Ohm
E=33.293
F=cfreq GHz
Term
Term2
Num=2
Z=50 Ohm
Term
Term3
Num=3
Z=50 Ohm
TLIN
TL15
Z=120 Ohm
E=34.875
F=cfreq GHz
TLIN
TL16
Z=20 Ohm
E=33.455
F=cfreq GHz
TLIN
TL12
Z=120 Ohm
E=71.478
F=cfreq GHz
TLIN
TL10
Z=20 Ohm
E=36.874
F=cfreq GHz
VAR
VAR1
cfreq=1
S-PARAMETERS
S_Param
SP1
Start=0 GHz
TLIN
TL11
Z=120 Ohm
E=71.478
F=cfreq GHz
TLIN
TL13
Z=20 Ohm
E=33.455
F=cfreq GHz
TLIN
TL14
Z=120 Ohm
E=34.875
F=cfreq GHz
Term
Term4
Num=4
Z=50 Ohm


dB(S(1,1))
dB(S(2,1))
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
freq, GHz


dB(S(1,1))
dB(S(2,1))
0
-20
-40
-60
0
1
2
freq, GHz


dB(S(3,3))
dB(S(4,3))
0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
freq, GHz


dB(S(3,3))
dB(S(4,3))
0
-20
-40
-60
0
1
2
freq, GHz