Formulas for Calculating Zoo and Zoe of Coupled Microstrip

Chris Haji-Michael
http://www.sunshadow.co.uk/chris.htm

This page calculates Zoo and Zoe for coupled microstrip. It uses the several equations combined together. The main equations are from "Accurate Wide-Range Design Equations for the Frequency Dependent Characteristics...", Kirschning, Jansen, IEEE MITT Jan 1984, Page 83 onwards.

The Effective Line Permittivity is from "Handbook of Microwave and Optical Components", Chang, Chapter 1, Table 1.16. This has a correction for line thickness.

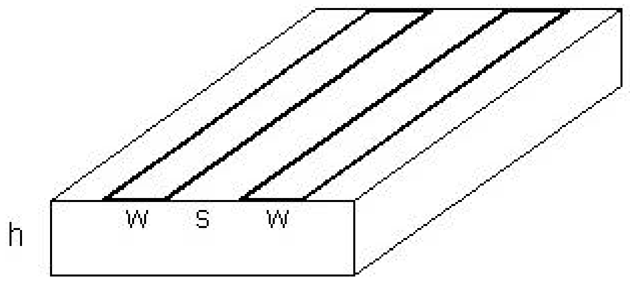
No calculation has been made for variation of impedance with frequency as the dimensions are so small that dispersion will not cause a problem, Refer to sheet for Zo for this equation. The results are compared to ADS linecalc and are pretty close but are 10% out for thick lines; thin lines match perfectly.

Definitions:

$\mu m := m \cdot 10^{-6}$

$c := 3 \times 10^{8} \cdot \frac{m}{s}$

$u := \frac{w}{h} \qquad g := \frac{s}{h} \qquad p := \frac{t}{h}$



Width Correction

$$u_{eff}(u,p) := \begin{cases} u + 1.25 \cdot \left(\frac{p}{\mathbf{p}}\right) \cdot \left(1 + \ln\left(\frac{4 \cdot \mathbf{p} \cdot u}{p}\right)\right) & \text{if } u < \frac{1}{2 \cdot \mathbf{p}} \\ u + 1.25 \cdot \left(\frac{p}{\mathbf{p}}\right) \cdot \left(1 + \ln\left(\frac{4 \cdot \mathbf{p} \cdot u}{p}\right)\right) & \text{if } u = \frac{1}{2 \cdot \mathbf{p}} \\ u + 1.25 \cdot \left(\frac{p}{\mathbf{p}}\right) \cdot \left(1 + \ln\left(\frac{2}{p}\right)\right) & \text{if } u > \frac{1}{2 \cdot \mathbf{p}} \end{cases}$$

Single Microstrip Line Effective Permittivity at dc

$$\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p) := \begin{cases} \frac{(\mathbf{e}_r + 1)}{2} + \frac{(\mathbf{e}_r - 1)}{2} \cdot \left[\left(1 + \frac{12}{u}\right)^{-0.5} + 0.04 \cdot \left(1 - \frac{1}{u}\right)^2\right] - \frac{(\mathbf{e}_r - 1)}{4.6} \cdot p \cdot \frac{1}{\sqrt{u}} & \text{if } u < 1 \\ \frac{(\mathbf{e}_r + 1)}{2} + \frac{(\mathbf{e}_r - 1)}{2} \cdot \left[\left(1 + \frac{12}{u}\right)^{-0.5} + 0.04 \cdot \left(1 - \frac{1}{u}\right)^2\right] - \frac{(\mathbf{e}_r - 1)}{4.6} \cdot p \cdot \frac{1}{\sqrt{u}} & \text{if } u = 1 \\ \frac{(\mathbf{e}_r + 1)}{2} + \frac{(\mathbf{e}_r - 1)}{2}\left[\left(1 + \frac{12}{u}\right)^{-0.5}\right] - \frac{(\mathbf{e}_r - 1)}{4.6} \cdot p \cdot \frac{1}{\sqrt{u}} & \text{if } u > 1 \end{cases}$$

$$\mathbf{e}_{eff.0}\left(4.1, \frac{7}{7}, \frac{0.1}{7}\right) = 2.97$$

Even Mode Effective Permittivity at dc

$$v(u,g) := u \cdot \frac{(20 + g^2)}{(10 + g^2)} + g \cdot \exp(-g)$$

$$b_e(\mathbf{e}_r) := 0.564 \cdot \left(\frac{\mathbf{e}_r - 0.9}{\mathbf{e}_r + 3.0}\right)^{0.053}$$

$$a_e(u,g) := 1 + \frac{1}{49} \cdot \ln\left[\frac{v(u,g)^4 + \left(\frac{v(u,g)}{52}\right)^2}{v(u,g)^4 + 0.432}\right] + \frac{1}{18.7} \cdot \ln\left[1 + \left(\frac{v(u,g)}{18.1}\right)^3\right]$$

$$\mathbf{e}_{eff.e.0}(\mathbf{e}_r,u,g) := \frac{\mathbf{e}_r+1}{2} + \frac{\mathbf{e}_r-1}{2}\cdot\left(1+\frac{10}{v(u,g)}\right)^{-a_e(u,g)\cdot b_e(\mathbf{e}_r)}$$

This agrees closely with HPADS for all values

$$\mathbf{e}_{eff.e.0}\left(4.1,\frac{5}{7},\frac{10}{7}\right) = 3.089$$

Impedance Zo

$$Zo(\mathbf{e}_r,u,p) := \begin{cases} \frac{60}{\sqrt{\mathbf{e}_{eff.e.0}(\mathbf{e}_r,u_{eff}(u,p),p)}}\cdot\ln\left(\frac{8}{u_{eff}(u,p)}+0.25\cdot u_{eff}(u,p)\right)\cdot ohm & \text{if } u_{eff}(u,p)<1 \\ \frac{60}{\sqrt{\mathbf{e}_{eff.e.0}(\mathbf{e}_r,u_{eff}(u,p),p)}}\cdot\ln\left(\frac{8}{u_{eff}(u,p)}+0.25\cdot u_{eff}(u,p)\right)\cdot ohm & \text{if } u_{eff}(u,p)=1 \\ \frac{120\cdot\mathbf{p}}{\sqrt{\mathbf{e}_{eff.e.0}(\mathbf{e}_r,u_{eff}(u,p),p)}}\cdot\frac{1}{(u_{eff}(u,p)+1.393+0.667\cdot\ln(1.444+u_{eff}(u,p)))}\cdot ohm & \text{if } u_{eff}(u,p)>1 \end{cases}$$

Compared to ADS this is 4% out for this narrow, thick tracks

$$Zo\left(4.1,\frac{5}{7},\frac{2}{7}\right) = 69.314\cdot ohm$$

Odd Mode Permittivity

$$b_o(\mathbf{e}_r) := \frac{0.747\cdot\mathbf{e}_r}{0.15+\mathbf{e}_r} \qquad c_o(\mathbf{e}_r,u) := b_o(\mathbf{e}_r) - (b_o(\mathbf{e}_r)-0.207)\cdot\exp(-0.414\cdot u) \qquad d_o(u) := 0.593+0.694\cdot\exp(-0.562\cdot u)$$

$$a_o(\mathbf{e}_r,u,p) := 0.7287\cdot\left(\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)-\frac{\mathbf{e}_r+1}{2}\right)\cdot(1-\exp(-0.179\cdot u))$$

$$\mathbf{e}_{eff.o.0}(\mathbf{e}_r,u,g,p) := \left[\left(\frac{\mathbf{e}_r+1}{2}+a_o(\mathbf{e}_r,u,p)\right)-\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)\right]\cdot\exp\left(-c_o(\mathbf{e}_r,u)\cdot g^{d_o(u)}\right)+\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)$$

Compared to ADS this is 22% out for this narrow and thick tracks. Wider and thinner tracks are in excellent agreement

$$\mathbf{e}_{eff.o.0}\left(4.1,\frac{5}{7},\frac{4}{7},\frac{2}{7}\right) = 2.586$$

Even Mode Characteristic Impedance

$$Q_1(u) := 0.8695\cdot u^{0.194} \qquad Q_2(g) := 1+0.7519\cdot g+0.189\cdot g^{2.31}$$

$$Q_3(g) := 0.1975+\left[16.6+\left(\frac{8.4}{g}\right)^6\right]^{-0.387}+\frac{1}{241}\cdot\ln\left[\frac{g^{10}}{1+\left(\frac{g}{3.4}\right)^{10}}\right]$$

$$Q_4(u,g) := 2\cdot\frac{Q_1(u)}{Q_2(g)}\cdot\left[u^{-Q_3(g)}\cdot\left[\exp(-g)\cdot u^{-Q_3(g)}+(2-\exp(-g))\right]\right]^{-1}$$

$$Z_{oe}(\mathbf{e}_r,u,p,g) := Zo(\mathbf{e}_r,u,p)\cdot\sqrt{\left(\frac{\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)}{\mathbf{e}_{eff.e.0}(\mathbf{e}_r,u,g)}\right)}\cdot\frac{1}{\left(1-\frac{\sqrt{\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)}\cdot Zo(\mathbf{e}_r,u,p)\cdot Q_4(u,g)}{377ohm}\right)}$$

Odd Mode Characteristic Impedance

$$Q_5(g) := 1.794 + 1.14 \cdot \ln\left(1 + \frac{0.638}{g + 0.517 \cdot g^{2.43}}\right)$$

$$Q_6(g) := 0.2305 + \frac{1}{281.3} \cdot \ln\left[\frac{g^{10}}{1 + \left(\frac{g}{5.8}\right)^{10}}\right] + \frac{\ln\left(1 + 0.598 \cdot g^{1.154}\right)}{5.1} \qquad Q_7(g) := \frac{10 + 190 \cdot g^2}{1 + 82.3 \cdot g^3}$$

$$Q_8(g) := \exp\left[-6.5 - 0.95 \cdot \ln(g) - \left(\frac{g}{0.15}\right)^5\right] \qquad Q_9(g) := \left(Q_8(g) + \frac{1}{16.5}\right) \cdot \ln\left(Q_7(g)\right)$$

$$Q_{10}(u,g) := \frac{Q_2(g) \cdot Q_4(u,g) - Q_5(g) \cdot \exp\left(\ln(u) \cdot Q_6(g) \cdot u^{-Q_9(g)}\right)}{Q_2(g)}$$

$$Z_{oo}(\mathbf{e}_r,u,p,g) := Zo(\mathbf{e}_r,u,p) \cdot \sqrt{\left(\frac{\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)}{\mathbf{e}_{eff.o.0}(\mathbf{e}_r,u,g,p)}\right)} \cdot \frac{1}{\left(1 - \frac{\sqrt{\mathbf{e}_{eff.0}(\mathbf{e}_r,u,p)} \cdot Zo(\mathbf{e}_r,u,p) \cdot Q_{10}(u,g)}{377 ohm}\right)}$$

This table shows values calculated by this MathCAD sheet and values calculated by ADS LineCalc. There is an excellent match for thin tracks

w(um)	s(um)	h(um)	t(um)	Er	Zo	Zlo	Zle	ADS Zo	ADS Zlo	ADS Zle
5	4	7	2	4.1	69.31	53.78	76.43	66.77	47.55	93.75
10	4	7	2	4.1	52.63	42.71	59.13	50.99	38.73	67.13
15	4	7	2	4.1	42.66	35.81	47.98	41.63	32.95	52.60
20	4	7	2	4.1	35.99	30.97	40.34	35.30	28.77	43.34
5	10	7	2	4.1	69.31	62.49	71.21	71.52	61.31	83.84
10	10	7	2	4.1	52.63	48.17	55.01	54.00	47.48	61.41
15	10	7	2	4.1	42.66	39.49	44.78	43.76	39.11	48.96
20	10	7	2	4.1	35.99	33.59	37.79	36.92	33.30	40.81
5	10	7	0.1	4.1	81.77	73.38	89.31	83.33	74.82	92.81
10	10	7	0.1	4.1	58.67	53.44	63.91	60.00	54.50	66.08
15	10	7	0.1	4.1	46.42	42.79	50.32	47.51	43.55	51.82
20	10	7	0.1	4.1	38.57	35.87	41.58	39.55	36.49	42.76

There are three impedances

Zo (**dielectric, with/h, thickness/h**) = impedance of the track as if it were on its own.

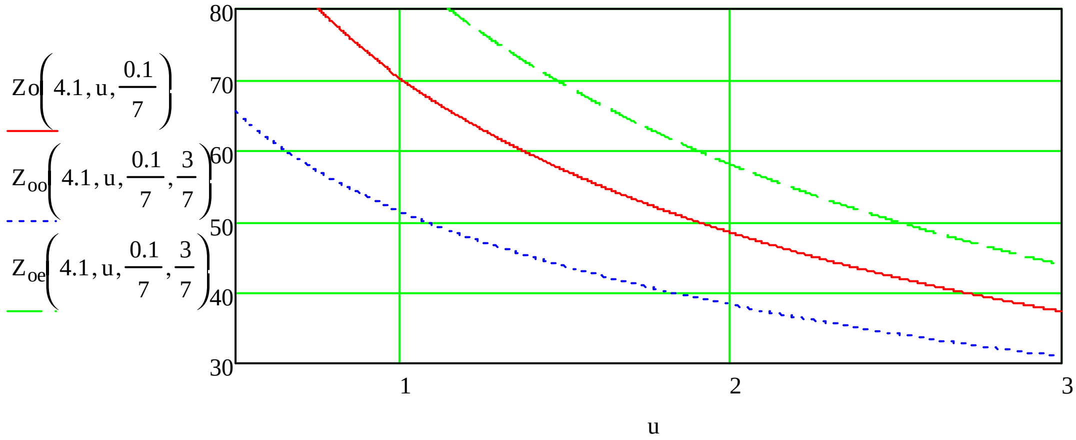
Zoo (**dielectric, with/h, thickness/h, gap/h**) = ODD mode impedance. This is the impedance of the single track if an inverted signal is put on the second track. This second out-of-phase signal increases the capacitance seen by the first track and **reduces** its impedance

Zoe (**dielectric, with/h, thickness/h, gap/h**) = EVEN mode impedance. This is the impedance of the single track if the same signal is put on the second track. This second in-phase signal reduces the capacitance seen by the first track and **increases** its impedance

As the gap between the two tracks incraeses then the impedances of Zoo and Zoe converge to Zo

$$Zo\left(4.1, \frac{5}{7}, \frac{0.1}{7}\right) = 81.771 \cdot ohm \qquad Z_{oo}\left(4.1, \frac{5}{7}, \frac{0.1}{7}, \frac{3}{7}\right) = 58.089 \cdot ohm \qquad Z_{oe}\left(4.1, \frac{5}{7}, \frac{0.1}{7}, \frac{3}{7}\right) = 100.18 \cdot ohm$$

Impedance as with track width changes



Impedance as the gap increases