Accurate Formulas for Calculating Zo of Microstrip with adjustment for line thickness and dispersion

This page calculates Zo for a microstrip line. The formulas are mostly from "Handbook of Microwave and Optical Components", editor K Chang, volume 1, publisher Wiley, table 1.16. There are many equations to calculate Zo but most of them do not properly adjust for line thickness. These results are compared with ADS linecalc results and are excellent. Dispersion is also compensated for.

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Definitions:

$\boldsymbol{\mu}\mathrm{m} := \mathrm{m}\cdot 10^{-6}$ $\quad c := 3\times 10^{8}\cdot\frac{\mathrm{m}}{\mathrm{s}}$ $\quad u := \frac{w}{h}$ $\quad g := \frac{s}{h}$ $\quad p := \frac{t}{h}$

Compensate for conductor thickness (t)

$$u_{eff}(u,p) := \begin{cases} u + 1.25\cdot\left(\frac{p}{\mathbf{p}}\right)\cdot\left(1 + \ln\left(\frac{4\cdot\mathbf{p}\cdot u}{p}\right)\right) & \text{if } u < \frac{1}{2\cdot\mathbf{p}} \\ u + 1.25\cdot\left(\frac{p}{\mathbf{p}}\right)\cdot\left(1 + \ln\left(\frac{4\cdot\mathbf{p}\cdot u}{p}\right)\right) & \text{if } u = \frac{1}{2\cdot\mathbf{p}} \\ u + 1.25\cdot\left(\frac{p}{\mathbf{p}}\right)\cdot\left(1 + \ln\left(\frac{2}{p}\right)\right) & \text{if } u > \frac{1}{2\cdot\mathbf{p}} \end{cases}$$

$$\mathbf{e}_{eff}(\mathbf{e}_r,u,p) := \begin{cases} \frac{(\mathbf{e}_r + 1)}{2} + \frac{(\mathbf{e}_r - 1)}{2}\cdot\left[\left(1 + \frac{12}{u}\right)^{-0.5} + 0.04\cdot(1-u)^2\right] - \frac{(\mathbf{e}_r - 1)}{4.6}\cdot p\cdot\frac{1}{\sqrt{u}} & \text{if } u < 1 \\ \frac{(\mathbf{e}_r + 1)}{2} + \frac{(\mathbf{e}_r - 1)}{2}\cdot\left[\left(1 + \frac{12}{u}\right)^{-0.5} + 0.04\cdot(1-u)^2\right] - \frac{(\mathbf{e}_r - 1)}{4.6}\cdot p\cdot\frac{1}{\sqrt{u}} & \text{if } u = 1 \\ \frac{(\mathbf{e}_r + 1)}{2} + \frac{(\mathbf{e}_r - 1)}{2}\left[\left(1 + \frac{12}{u}\right)^{-0.5}\right] - \frac{(\mathbf{e}_r - 1)}{4.6}\cdot p\cdot\frac{1}{\sqrt{u}} & \text{if } u > 1 \end{cases}$$

$$Zo(\mathbf{e}_r,u,p) := \begin{cases} \frac{60}{\sqrt{\mathbf{e}_{eff}(\mathbf{e}_r,u,p)}}\cdot\ln\left(\frac{8}{u_{eff}(u,p)} + 0.25\cdot u_{eff}(u,p)\right)\cdot\mathrm{ohm} & \text{if } u_{eff}(u,p) < 1 \\ \frac{60}{\sqrt{\mathbf{e}_{eff}(\mathbf{e}_r,u,p)}}\cdot\ln\left(\frac{8}{u_{eff}(u,p)} + 0.25\cdot u_{eff}(u,p)\right)\cdot\mathrm{ohm} & \text{if } u_{eff}(u,p) = 1 \\ \frac{120\cdot\mathbf{p}}{\sqrt{\mathbf{e}_{eff}(\mathbf{e}_r,u,p)}}\cdot\frac{1}{(u_{eff}(u,p) + 1.393 + 0.667\cdot\ln(1.444 + u_{eff}(u,p)))}\cdot\mathrm{ohm} & \text{if } u_{eff}(u,p) > 1 \end{cases}$$

Dispersion

The effective permitivity is influenced by the maximum frequency limit of the microstrip. This little-known adjustment is from Pramanick & Bhartia

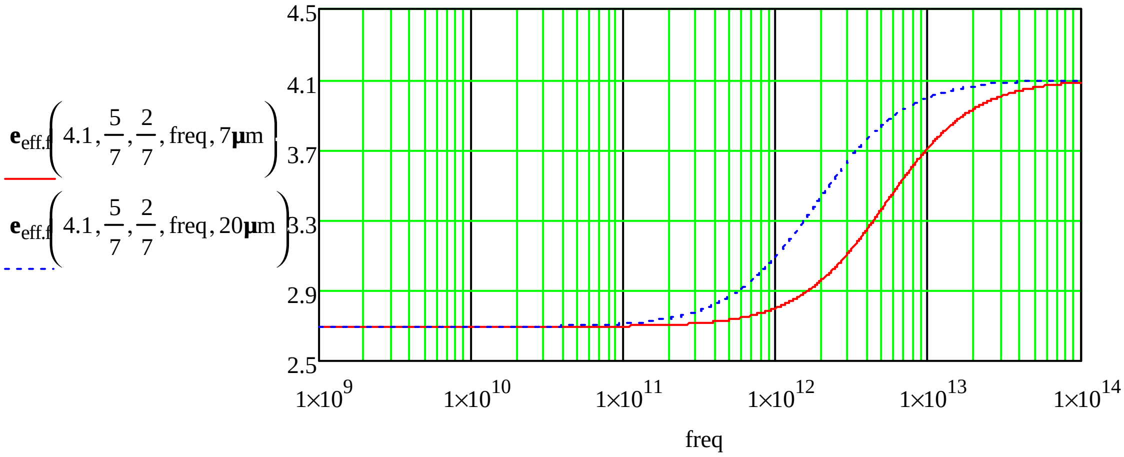
$$F(\mathbf{e}_r,u,p,freq,h) := \frac{\left(4\cdot h\cdot freq\cdot\sqrt{\mathbf{e}_r - 1}\right)}{c}\cdot\left[0.5 + \left(1 + 2\cdot\log\left(1 + u_{eff}(u,p)\right)\right)^2\right]$$

$$F\left(4.1, \frac{10}{7}, \frac{2}{7}, 1\cdot 10^{9}\mathrm{Hz}, 7\boldsymbol{\mu}\mathrm{m}\right) = 6.648\times 10^{-4}$$

The permittivity is plotted for typical IC dimensions against frequency. It can be seen that dispersion need only be considered above 1E12 Hz

$$\mathbf{e}_{eff.f}(\mathbf{e}_r, u, p, freq, h) := \left(\frac{\sqrt{\mathbf{e}_r} - \sqrt{\mathbf{e}_{eff}(\mathbf{e}_r, u, p)}}{1 + 4 \cdot F(\mathbf{e}_r, u, p, freq, h)^{-1.5}} + \sqrt{\mathbf{e}_{eff}(\mathbf{e}_r, u, p)} \right)^2$$

$$Z_{o.f}(\mathbf{e}_r, u, p, freq, h) := Zo(\mathbf{e}_r, u, p) \cdot \left(\frac{\mathbf{e}_{eff.f}(\mathbf{e}_r, u, p, freq, h) - 1}{\mathbf{e}_{eff}(\mathbf{e}_r, u, p) - 1} \cdot \sqrt{\frac{\mathbf{e}_{eff}(\mathbf{e}_r, u, p)}{\mathbf{e}_{eff.f}(\mathbf{e}_r, u, p, freq, h)}} \right)$$



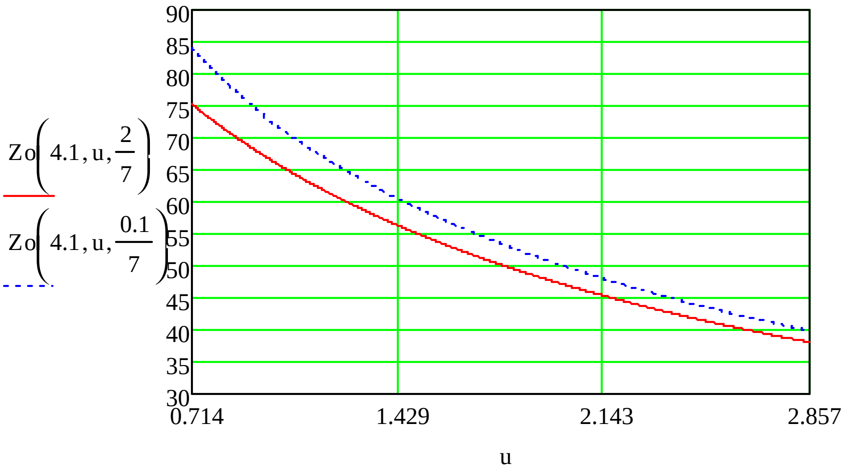
The value of Zo compares well with values calculated by HPADS and APLAC.

$Zo\left(4.1, \frac{5}{7}, \frac{2}{7}\right) = 75.26 \cdot ohm$ $\mathbf{e}_{eff}\left(4.1, \frac{5}{7}, \frac{2}{7}\right) = 2.695$

$p := \frac{t}{h}$

$u := \frac{w}{h}$

$g := \frac{s}{h}$



w (um)	h (um)	t (um)	Er	Zo	Eeff	ADS Zo	ADS Eeff
5	7	2	4.1	75.26	2.70	75.690	2.685
10	7	2	4.1	56.33	2.89	56.280	2.881
15	7	2	4.1	45.35	3.02	45.270	3.013
20	7	2	4.1	38.08	3.12	38.020	3.113
5	7	0.1	4.1	83.94	2.91	84.320	2.891
10	7	0.1	4.1	60.54	3.05	60.540	3.036
15	7	0.1	4.1	48.00	3.15	47.840	3.142
20	7	0.1	4.1	39.90	3.22	39.810	3.225