FM Range Calculation

This sheet is to estimate of the range that can be expected from an FM or ASK modulated system.

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It uses a modified-version of the Friss transmission equation. It starts by calculating the minimum signal strength in dBm that is required in the receiver. It uses three probability-error-function curves, the first one is for FSK with average white gaussian noise (AWGN), a second one is form ASK and the third is for FSK with Raleigh fading.

Many of the RF equations are adapted from Pozar, Microwave & RF Design of Wireless Systems. The propagation equations were found on http://www.sss-mag/indoor.html and http://people.deas.harvard.edu/~jones/es151/prop_models/propagation.html. The error function calculations are from "Wireless Communications", by Andrea Goldsmith.

$x := -5, -4.9 .. 25$

Probability Error Function for coherent 2FSK

The probability error function curve for coherent FSK can be used to calculate BER for different Eb/No. This equation is found in "An Intro to Analogue & Digital Communications" by Simon Haykin and Andrea Goldsmith equation 5.41 & table 6.1 where x is in dB.

$x := 1$ Given $PEF_{2fsk} = 0.5\,\mathrm{erfc}\left(\sqrt{\frac{10^{\frac{x}{10}}}{2}}\right)$ $EbNo\_2FSK\_dB(PEF_{2fsk}) := \mathrm{Find}(x)$

$EbNo\_2FSK\_dB(0.1) = 2.155$

$EbNo\_2FSK\_dB(0.01) = 7.333$

$PEF_{2fsk}(x) := 0.5\,\mathrm{erfc}\left(\sqrt{\frac{10^{\frac{x}{10}}}{2}}\right)$

Probability Error Function for coherent ASK

The probability error function curve for 2 level pulsed amplitude modulation (also called amplitude shift keying or on/off keying). This equation is from Andrea Goldsmith 5.41 & table 6.1 for MPAM where x is in dB

$x := 1$ Given $PEF_{2ask} = \frac{2}{2\cdot\log(2,2)}\,\mathrm{erfc}\left(\sqrt{\frac{10^{\frac{x}{10}}}{2}}\right)$ $EbNo\_2ASK\_dB(PEF_{2ask}) := \mathrm{Find}(x)$

$EbNo\_2ASK\_dB(0.001) = 10.345$

$PEF_{2ask}(x) := \frac{2}{2\cdot\log(2,2)}\,\mathrm{erfc}\left(\sqrt{\frac{10^{\frac{x}{10}}}{2}}\right)$

Probability Error Function for coherent 2FSK with fading

A real signal is likely to be subjected to fading of which the most severe version is **Raleigh fading**. This is best represented as an alternate Error Function Curve and is calculated here according to "Wireless Communications", by Andrea Goldsmith, equation 6.59.

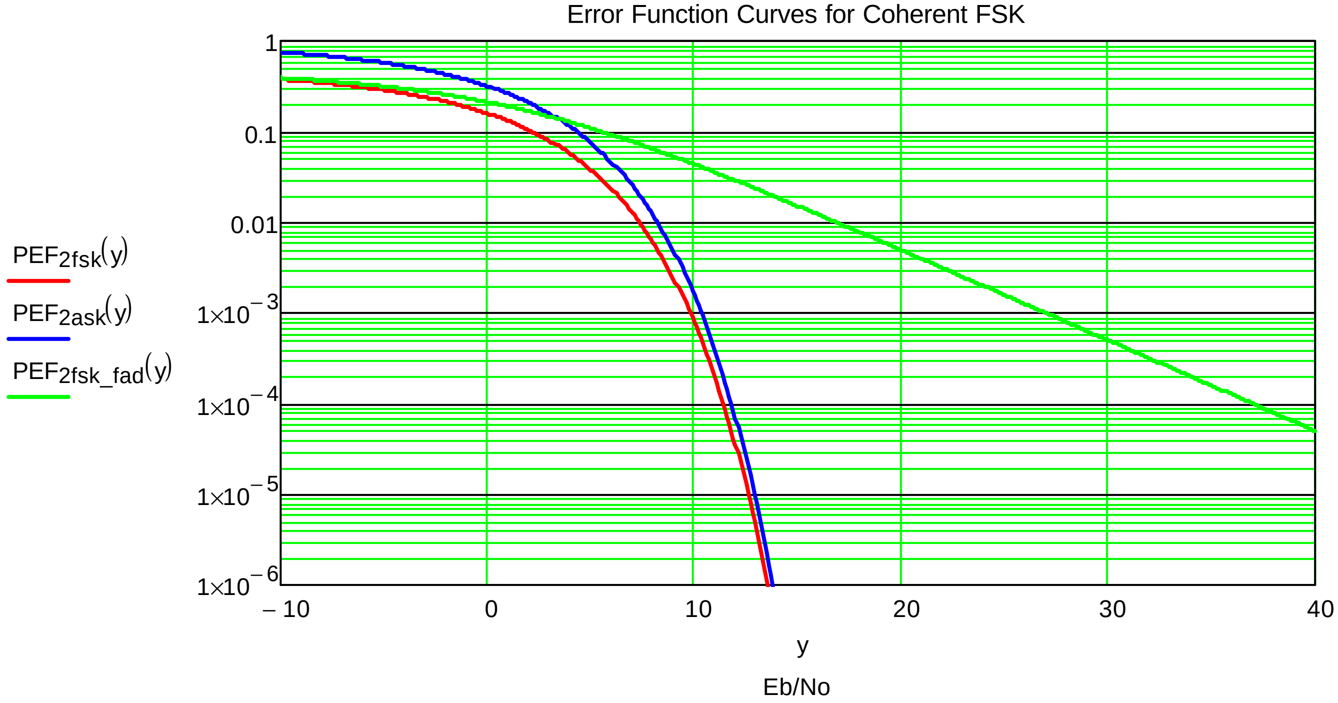
$y := 1$ Given $PEF_{2fsk\_fad} = 0.5\left(1 - \sqrt{\dfrac{10^{\frac{y}{10}}}{2 + 10^{\frac{y}{10}}}}\right)$

$EbNo\_2FSK\_fad\_dB\left(PEF_{2fsk\_fad}\right) := Find(y)$

$EbNo\_2FSK\_fad\_dB(0.1) = 5.509$

$PEF_{2fsk\_fad}(x) := 0.5\left(1 - \sqrt{\dfrac{10^{\frac{x}{10}}}{2 + 10^{\frac{x}{10}}}}\right)$

$y := -10, -9.9 .. 40$



RED curve is with average white gaussian noise for 2FSK (frequency shift keying)

BLUE curves with average white gaussian noise for ASK (amplitude keying or on/off keying)

GREEN curve is with Raleigh fading for 2FSK

Required receiver bandwidth for 2FSK

FM systems require a receive bandwidth which depends on the deviation and bit period. This is called the Carson's BW. More advanced FM receivers that use a PLL to track the input signal effectivly reduces this bandwidth. Remember, the bigger the bandwidth the more noise is seen by the receiver and the worse the receive sensitivity.

$$BT(fm\_dev, bit\_period) := fm\_dev \cdot bit\_period \qquad Carsons\_BW(fm\_dev, bit\_period) := 2 \cdot \left(\frac{1}{2 \cdot bit\_period} + fm\_dev \right)$$

Limit of sensitivity for 2FSK in AWGN

The limit of sensitivity for FM systems uses the PEF curve and adds to this the noise floor (-174dBm/Hz) and noise you would get with the bandwidth you specify, as well as the noise-figure of the receiver. There is also an adjustment for the BT which is a measure of how large the eye. A low data rate has a large eye size and is easier to receive.

In advanced receivers that use a PLL as the demodulator, this effectivly reduces the noise bandwith. The IF bandwidth needs to be greater than the Carson's bandwidth. The results here can be directly compared to the receiver measurements.

$$min\_detect\_signal\_fsk\_dBm(BER, fm\_dev, bit\_period, rx\_NF\_dB, BW) := -174 + 10 \cdot \log\left(\frac{BW}{Hz}\right) + EbNo\_2FSK\_dB(BER) + rx\_NF\_dB - 10\log(2\,BT(fm\_dev, bit\_period))$$

$$min\_detect\_signal\_fsk\_W(BER, fm\_dev, bit\_period, rx\_NF\_dB, BW) := 10^{\frac{min\_detect\_signal\_fsk\_dBm(BER, fm\_dev, bit\_period, rx\_NF\_dB, BW)}{10}} \cdot mW$$

$$Carsons\_BW\left(20kHz, \frac{s}{2 \cdot 20000}\right) = 80 \cdot kHz \qquad min\_detect\_signal\_fsk\_dBm\left(0.001, 20kHz, \frac{s}{2 \cdot 20000}, 5, 100kHz\right) = -109$$

$$Carsons\_BW\left(30kHz, \frac{s}{2 \cdot 10000}\right) = 80 \cdot kHz \qquad min\_detect\_signal\_fsk\_dBm\left(0.001, 30kHz, \frac{s}{2 \cdot 10000}, 5, 100kHz\right) = -114$$

$$Carsons\_BW\left(35kHz, \frac{s}{2 \cdot 5000}\right) = 80 \cdot kHz \qquad min\_detect\_signal\_fsk\_dBm\left(0.001, 35kHz, \frac{s}{2 \cdot 5000}, 5, 100kHz\right) = -118$$

$$Carsons\_BW\left(37.5kHz, \frac{s}{2 \cdot 2500}\right) = 80 \cdot kHz \qquad min\_detect\_signal\_fsk\_dBm\left(0.001, 37.5kHz, \frac{s}{2 \cdot 2500}, 5, 100kHz\right) = -121$$

These figures show that the sensitivity and BW for different deviations and bit period. The 2 for the bit period because the system is manchester encoded.

The sensitivity limit is for AWGN

Note the BW is the noise bandwidth and not the 3dB bandwidth

Limit of sensitivity for 2FSK in Raleigh Fading

A similar analysis can be carried out for the fading environment, but of course we cannot measure this directly.

$$min\_detect\_signal\_fskfad\_dBm(BER, fm\_dev, bit\_period, rx\_NF\_dB, BW) := -174 + 10\cdot\log\left(\frac{BW}{Hz}\right) + EbNo\_2FSK\_fad\_dB(BER) + rx\_NF\_dB - 10\log(2\,BT(fm\_dev, bit\_perioc$$

$$min\_detect\_signal\_fskfad\_W(BER, fm\_dev, bit\_period, rx\_NF\_dB, BW) := 10^{\frac{min\_detect\_signal\_fskfad\_dBm(BER, fm\_dev, bit\_period, rx\_NF\_dB, BW)}{10}}\cdot mW$$

$$min\_detect\_signal\_fskfad\_dBm\left(0.001, 37.5kHz, \frac{s}{2\cdot 2500}, 7, 100kHz\right) = -102$$

Required receiver bandwidth for ASK

An ASK system requires an IF bandwith that is equal to twice the maximum data rate. This is less than the required BW for FSK, where the deviation is very much greater than the modulation, then ASK gives a big reduction in the required RX BW which will help with limiit of sensitivity.

$$ASK\_BW(bit\_period) := 2\cdot\left(\frac{1}{2\cdot bit\_period}\right) \qquad ASK\_BW\left(\frac{s}{2\cdot 20000}\right) = 40\cdot kHz$$

Limit of sensitivity for ASK in AWGN

A similar analysis can be carried out for the AWGN environment with ASK

$$min\_detect\_signal\_ask\_dBm(BER, bit\_period, rx\_NF\_dB, BW) := -174 + 10\cdot\log\left(\frac{BW}{Hz}\right) + EbNo\_2ASK\_dB(BER) + rx\_NF\_dB$$

$$min\_detect\_signal\_ask\_W(BER, bit\_period, rx\_NF\_dB, BW) := 10^{\frac{min\_detect\_signal\_ask\_dBm(BER, bit\_period, rx\_NF\_dB, BW)}{10}}\cdot mW$$

min_detect_signal_ask_W(BER, bit_period, rx_NF_dB, BW)

$\text{ASK\_BW}\left(\frac{s}{2\cdot 20000}\right) = 40\cdot\text{kHz}$ $\text{min\_detect\_signal\_ask\_dBm}\left(0.001, \frac{s}{2\cdot 20000}, 5, 50\text{kHz}\right) = -112$

$\text{ASK\_BW}\left(\frac{s}{2\cdot 10000}\right) = 20\cdot\text{kHz}$ $\text{min\_detect\_signal\_ask\_dBm}\left(0.001, \frac{s}{2\cdot 10000}, 5, 50\text{kHz}\right) = -112$

$\text{ASK\_BW}\left(\frac{s}{2\cdot 5000}\right) = 10\cdot\text{kHz}$ $\text{min\_detect\_signal\_ask\_dBm}\left(0.001, \frac{s}{2\cdot 5000}, 5, 20\text{kHz}\right) = -116$

$\text{ASK\_BW}\left(\frac{s}{2\cdot 2500}\right) = 5\cdot\text{kHz}$ $\text{min\_detect\_signal\_ask\_dBm}\left(0.001, \frac{s}{2\cdot 2500}, 5, 10\text{kHz}\right) = -119$

These figures show that the sensitivity and BW for different data rates. The 2 in the data rate for the bit period because the system is manchester encoded.

The sensitivity limit is for AWGN

Note the BW is the noise bandwidth and not the 3dB bandwidth

This plot shows the receive sensitivity curves for FSK (with and without fading) and for ASK without fading

This part is a bit confusing, it plots the PEF curves in a complex way because it sweeps signal stength on the x-axis to get the Bit Error Rate (BER) rather than the other way around. It does this by having a starting BER to make the convergence work, I have put 0.01 here but it does not matter.

A useful plot to test receiver in 2FSK is to use the **RED** curve.
A useful plot to test receiver in 2ASK is to use the **BLUE** curve.

offset_fsk(BER, fm_dev, bit_period, rx_NF_dB, BW) := min_detect_signal_fsk_dBm(BER, fm_dev, bit_period, rx_NF_dB, BW) − EbNo_2FSK_dB(BER)

offset_ask(BER, bit_period, rx_NF_dB, BW) := min_detect_signal_ask_dBm(BER, bit_period, rx_NF_dB, BW) − EbNo_2ASK_dB(BER)

$\text{offset\_fsk}\left(0.1, 20\text{kHz}, \frac{s}{2\cdot 20000}, 5, 100\text{kHz}\right) = -119.0$ $\text{offset\_ask}\left(0.1, \frac{s}{2\cdot 20000}, 5, 40\text{kHz}\right) = -123.0$

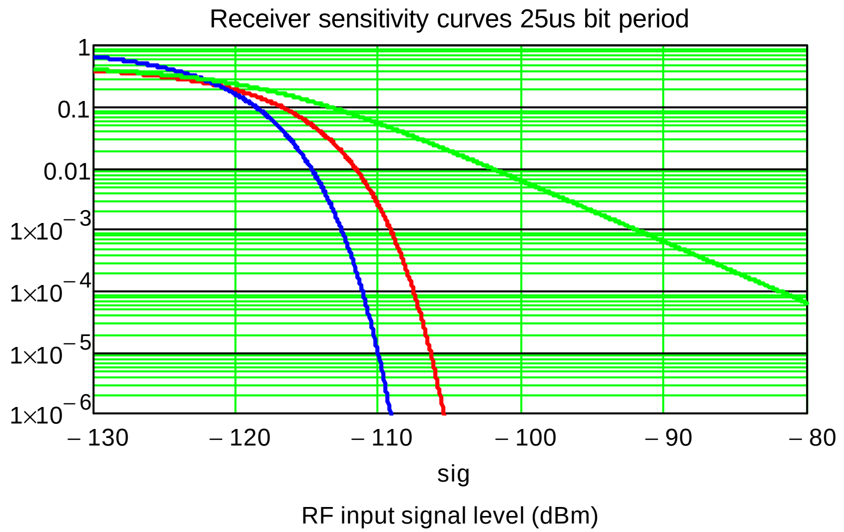
RED curve is receiver sensitivity curves for FSK 20KHz dev, 25us bit period, 5dB NF with average white gaussian noise
BLUE curve is receiver sensitivity curves for ASK, 25us bit period, 5dB NF with average white gaussian noise.
GREEN curve is receiver sensitivity curves for FSK 20KHz dev, 25us bit period, 5dB NF with Raleigh Fading

Note, the ASK curve uses a reduced BW

PEF2fsk(sig−offset_fsk(0.01, 20kHz, 25μs, 5, 100kHz))

PEF2ask(sig−offset_ask(0.01, 25μs, 5, 40kHz))

PEF2fsk_fad(sig−offset_fsk(0.01, 20kHz, 25μs, 5, 100kHz))



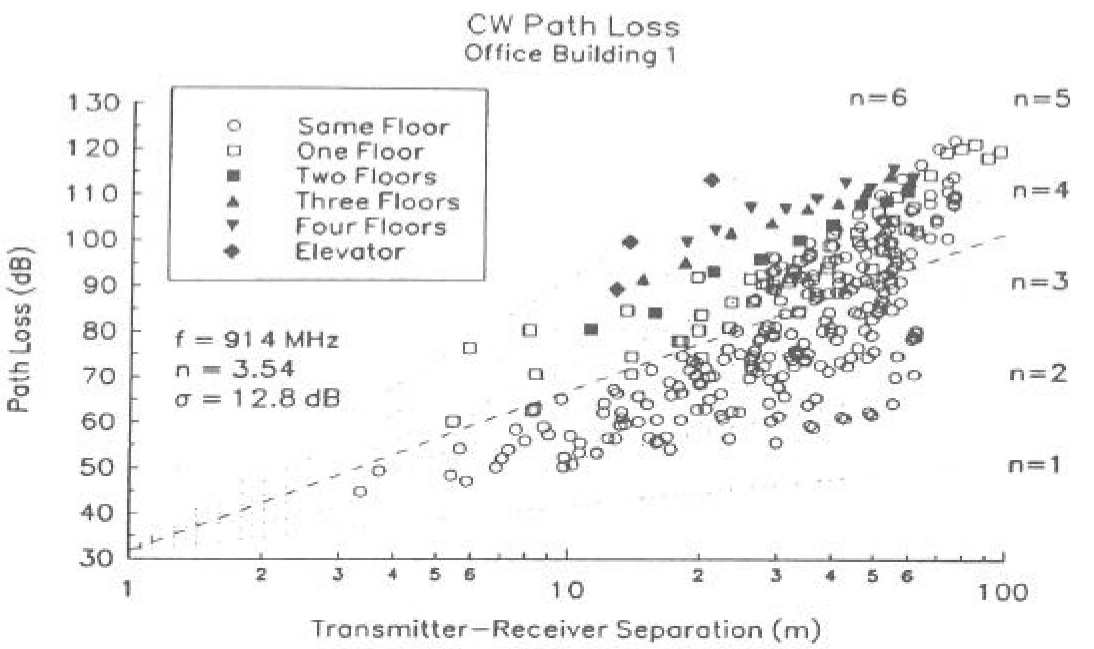
Calculate the expected range

Using a modified form of the Friss equation from **Rappaport & Theodore**. N is the propagation coefficient, refer to the picture below. These range calculations use both the **Raleigh** fading model and **non Raleigh** fading.

freq := 860·MHz   n := 2.5

The Friis equation is modified according plots from (RT) **Rappaport, Theodore**., Wireless Communications - Principles & Practice, IEEE Press, 1996.

In this sheet I have set **n=2.5** as communication is same floor.



$$\text{antenna\_gain}(\text{antenna\_gain\_dB}) := 10^{\frac{\text{antenna\_gain\_dB}}{10}} \qquad \text{tx\_power\_W}(\text{tx\_power\_dBm}) := 10^{\frac{\text{tx\_power\_dBm}}{10}} \cdot \text{mW}$$

$$\text{RangeRT\_fad}(\text{tx\_power\_dBm}, \text{BER}, \text{fm\_dev}, \text{bit\_period}, \text{rx\_NF\_dB}, \text{antenna\_gain\_dB}, \text{BW}) := \frac{3 \cdot 10^{8} \frac{\text{m}}{\text{s}}}{4 \cdot \mathbf{p} \cdot \text{freq}} \left(\frac{\text{antenna\_gain}(\text{antenna\_gain\_dB})^{2} \cdot \text{tx\_power\_W}(\text{tx\_power\_dBm})}{\text{min\_detect\_signal\_fskfad\_W}(\text{BER}, \text{fm\_dev}, \text{bit\_period}, \text{rx\_NF\_dB}, \text{BW})} \right)$$

$$\text{RangeRT}(\text{tx\_power\_dBm}, \text{BER}, \text{fm\_dev}, \text{bit\_period}, \text{rx\_NF\_dB}, \text{antenna\_gain\_dB}, \text{BW}) := \frac{3 \cdot 10^{8} \frac{\text{m}}{\text{s}}}{4 \cdot \mathbf{p} \cdot \text{freq}} \left(\frac{\text{antenna\_gain}(\text{antenna\_gain\_dB})^{2} \cdot \text{tx\_power\_W}(\text{tx\_power\_dBm})}{\text{min\_detect\_signal\_fsk\_W}(\text{BER}, \text{fm\_dev}, \text{bit\_period}, \text{rx\_NF\_dB}, \text{BW})} \right)^{\frac{1}{n}}$$

The range is calculated with the TX power in dBm and required BER for **Rappaport & Theodore,** with Raleigh fading on the left, non Raleigh on the right. I would expect communication to work reliably to the Raleigh limit, with decreasing performance to the higher limit

Summary

RangeRT(txpower(dBm), BER, dev, bitperiod, NF, Antennagain(dB), BW)

This range is calculated with the TX power in dBm for **Rappaport & Theodore.** Note, the bit period is divided by 2 as the data is Manchester encoded over the air. AWGN on the left, Raleigh fading values on the right. The range is between the two values.

Range 1
6dBm TX, 5dB NF, 20kHz dev, 20kHz air data rate, average antenna gain -15dB,
200bits without error, no FEC.

$$\text{RangeRT}\left(6, 0.005, 20\text{kHz}, \frac{\text{s}}{2 \cdot 20000}, 5, -15, 100\text{kHz}\right) = 82\,\text{m} \qquad \text{RangeRT\_fad}\left(6, 0.005, 20\text{kHz}, \frac{\text{s}}{2 \cdot 20000}, 5, -15, 100\text{kHz}\right) = 28\,\text{r}$$

Range 2
TX 6dBm TX, 1:10 FEC, reduce the data rate by quarter, no Manchester.

$$\text{RangeRT}\left(6, 0.1, 20\text{kHz}, \frac{\text{s}}{5000}, 5, -15, 50\text{kHz}\right) = 435\,\text{m} \qquad \text{RangeRT\_fad}\left(6, 0.1, 20\text{kHz}, \frac{\text{s}}{5000}, 5, -15, 50\text{kHz}\right) = 320\,\text{m}$$

Improvements. The ASK has not been included in the range calculation because this work needs to be double checked and needs to be compared to measurements. The FSK curves and plots are correct and have been checked and are in the range calculation. This work is ongoing but provides quite a useful sheet as it is.